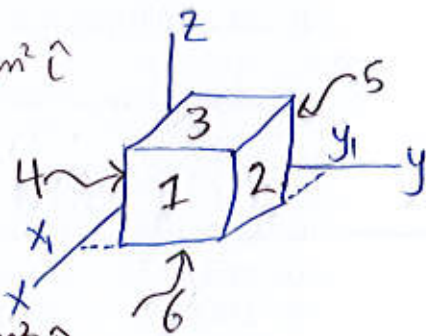


23-23 A closed Gaussian surface in the shape of a cube of edge length 2.00 m, with one corner at $x_1 = 5.00$ m, $y_1 = 4.00$ m. The cube lies in a region where the electric field vector is given by $\vec{E} = +23.0\hat{i} - 2.00y^2\hat{j} - 16.0\hat{k}$ N/C, with y in meters. What is the net charge contained by the cube?

$$q_{enc} = \epsilon_0 \Phi, \quad \Phi = \oint \vec{E} \cdot d\vec{A}$$

\Rightarrow Cube face (1) (located at $x = 5.00$ m), $d\vec{A} = 4\text{ m}^2 \hat{i}$

$$\begin{aligned} \Phi_1 &= (+23.0\hat{i} - 2.00y^2\hat{j} - 16.0\hat{k}) \cdot 4\hat{i} \\ &= +92 \text{ N}\cdot\text{m}^2/\text{C} \end{aligned}$$



\Rightarrow Cube face (2), located at $y = 4.00$ m, $d\vec{A} = 4\text{ m}^2 \hat{j}$

$$\Phi_2 = -2(4)^2 \times 4 = -128 \text{ N}\cdot\text{m}^2/\text{C}$$

\Rightarrow Cube face (3), located at $z = 2.0$ m, $d\vec{A} = 4\text{ m}^2 \hat{k}$

$$\Phi_3 = -16 \times 4 = -64 \text{ N}\cdot\text{m}^2/\text{C}$$

\Rightarrow Cube face (4), located at $y = 2$ m, $d\vec{A} = -4\text{ m}^2 \hat{j}$

$$\Phi_4 = -2(2)^2 \times -4 = +32 \text{ N}\cdot\text{m}^2/\text{C}$$

\Rightarrow Cube face (5), located at $x = 3.0$ m, $d\vec{A} = -4\text{ m}^2 \hat{i}$

$$\Phi_5 = 23 \times -4 = -92 \text{ N}\cdot\text{m}^2/\text{C}$$

\Rightarrow Cube face (6), located at $z = 0$, $d\vec{A} = -4\text{ m}^2 \hat{k}$

$$\Phi_6 = -16 \times -4 = +64 \text{ N}\cdot\text{m}^2/\text{C}$$

$$\Rightarrow \Phi_{total} = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + \Phi_5 + \Phi_6 = -96 \frac{\text{N}\cdot\text{m}^2}{\text{C}}$$

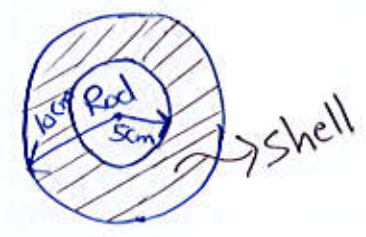
$$q_{enc} = \epsilon_0 \Phi = 8.85 \times 10^{-12} \times -96 = -8.496 \times 10^{-10} \text{ C}$$

$$q_{enc} = -0.85 \text{ nC}$$

23-4 A charge of uniform linear density 1.5 nC/m is distributed along a long, thin, non-conducting rod. The rod is coaxial with a long conducting cylindrical shell (inner radius = 5.0 cm , outer radius = 10.0 cm). The net charge on the shell is zero. a) What is the magnitude of the electric field 15.0 cm from the axis of the shell? What is the surface charge density on the b) inner and c) outer surface of the shell?

\Rightarrow a) $E = \frac{\lambda}{2\pi\epsilon_0 r}$ (Line of charge)

$$E = \frac{1.5 \times 10^{-9}}{2 \times \pi \times 8.85 \times 10^{-12} \times 0.15} = 180 \text{ N/C}$$



b) The net charge on the shell is zero and the shell is conducting. So the electric field is zero inside the conductor

\Rightarrow inner surface has negative linear density $(-\lambda)$
 outer surface has positive linear density $(+\lambda)$
 where λ is the linear density of the rod = 1.5 nC/m

* Surface charge density on the inner of the shell

$$\sigma_{\text{inner}} = \frac{-q}{\text{area}} = \frac{-q}{2\pi r L} = \frac{-\lambda}{2\pi r}$$

$$\sigma_{\text{inner}} = \frac{-1.5 \times 10^{-9}}{2 \times \pi \times 0.05} = -4.78 \frac{\text{nC}}{\text{m}^2}$$

$$\lambda = \frac{q}{L}$$

* Surface charge density of the outer of the shell

$$\sigma_{\text{outer}} = \frac{+q}{\text{Area}} = \frac{+q}{2\pi r L} = \frac{+\lambda}{2\pi r}$$

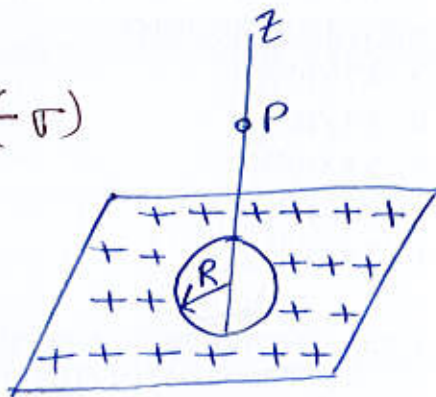
$$\sigma_{\text{outer}} = \frac{+1.5 \times 10^{-9}}{2\pi \times 0.1} = +2.39 \frac{\text{nC}}{\text{m}^2}$$

23-8 | A small circular hole of radius $R = 1.30$ cm has been cut in the middle of an infinite, flat, non-conducting surface that has uniform charge density $\sigma = 4.50$ pC/m². A z axis, with its origin at the hole's center, is perpendicular to the surface. In unit vector notation, what is the electric field at point P at $z = 2.56$ cm?

Let the circular hole has charge density $(-\sigma)$

$$\Rightarrow \vec{E}_1 (\text{infinite, flat, non-conducting sheet}) = \frac{\sigma}{2\epsilon_0} \hat{k}$$

$$\Rightarrow \vec{E}_2 (\text{circular hole}) = -\frac{\sigma}{2\epsilon_0} \left[1 - \frac{z}{\sqrt{z^2 + R^2}} \right] \hat{k}$$



$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\sigma z}{2\epsilon_0} \left[\frac{1}{\sqrt{z^2 + R^2}} \right] \hat{k}$$

$$\vec{E} = \frac{4.5 \times 10^{-12} \times 2.56 \times 10^{-2}}{2 \times 8.85 \times 10^{-12}} \left[\frac{1}{\sqrt{(2.56 \times 10^{-2})^2 + (1.3 \times 10^{-2})^2}} \right] \hat{k}$$

$$\boxed{\vec{E} = 0.227 \frac{N}{C} \hat{k}}$$

23-50 Flux and conducting shells. A charged particle is held at the center of two concentric conducting spherical shells. The below figure gives the net flux Φ through a Gaussian sphere centered on the particle, as a function of the radius r of the sphere. The scale of the vertical axis is set by $\Phi_s = 10 \times 10^5 \text{ N}\cdot\text{m}^2/\text{C}$. What are a) the charge of the central particle and the net charges of shell A and B?

\Rightarrow Gauss' Law $q_{\text{enc}} = \epsilon_0 \Phi$

a) $\Phi = -18 \times 10^5 \frac{\text{N}\cdot\text{m}^2}{\text{C}}$ for small r "Gaussian surface surrounds only the particle"

$q_{\text{enc}} = 8.85 \times 10^{-12} \times -18 \times 10^5 = -1.593 \times 10^{-5} \text{ C}$

$q_{\text{central particle}} = -16 \mu\text{C}$

b) $\Phi = +8.0 \times 10^5 \frac{\text{N}\cdot\text{m}^2}{\text{C}} \Rightarrow q'_{\text{enc}} = +7.08 \mu\text{C}$ "Gaussian surface surrounds the particle and shell A"

$q_{\text{shell A}} = q'_{\text{enc}} - q_{\text{central particle}}$

$= (+7.08 + 16) \mu\text{C} = 23.08 \mu\text{C} = q_{\text{shell A}}$

c) $\Phi = -4.0 \times 10^5 \frac{\text{N}\cdot\text{m}^2}{\text{C}} \Rightarrow q''_{\text{enc}} = -3.54 \mu\text{C}$ "Gaussian surface surrounds the particle and shell A and B"

$q''_{\text{enc}} \equiv \text{total enclosed charge}$

$q_{\text{shell B}} = (-3.54 - 23.08 - -16) \mu\text{C}$

$q_{\text{shell B}} = -10.62 \mu\text{C}$

6] Magnitude of the electric field inside and outside a sphere with a positive charge distributed uniformly through out its volume. The scale of the vertical axis is set by $E_s = 10 \times 10^7 \text{ N/C}$

a) What is the charge on the sphere?

$$E(r=2\text{cm} \equiv \text{radius of sphere}) = 10 \times 10^7 \text{ N/C}$$

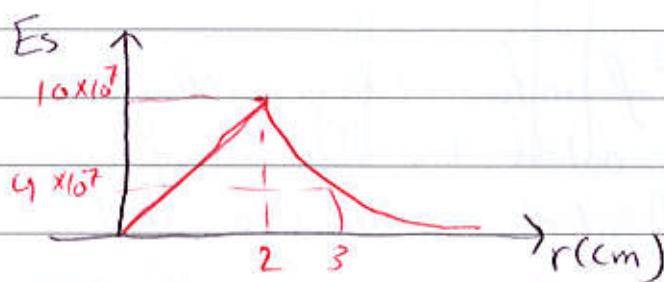
$$E = k \frac{q}{R^2} \Rightarrow q = \frac{10 \times 10^7 \times (2 \times 10^{-2})^2}{9 \times 10^9}$$

$$q \approx 4.44 \mu\text{C}$$

b) What is the field magnitude at $r=8.0\text{m}$?

$r=8.0\text{m}$ outside the sphere

$$E = \frac{kq}{r^2} = \frac{9 \times 10^9 \times 4.44 \times 10^{-6}}{(8)^2} = 6.25 \times 10^2 \text{ N/C}$$



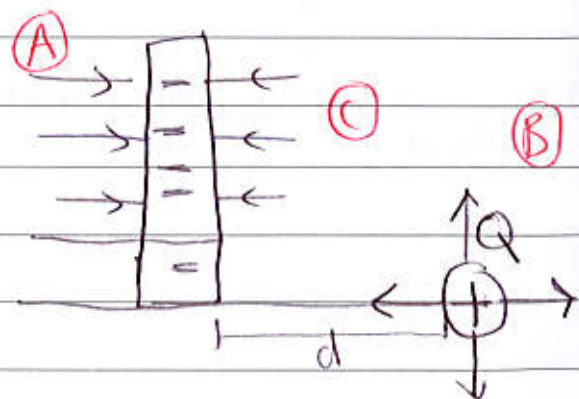
$r \propto R$, E is a straight line

$$E = \frac{kq}{R^3} r$$

10] Very large non-conducting sheet that has a uniform surface charge density $\sigma = -2.00 \mu\text{C}/\text{m}^2$ with a particle of charge $Q = 8 \mu\text{C}$ at distance d from the sheet. Both are fixed in place. if $d = 0.2\text{m}$

a) at what +ve and b) -ve coordinate on x-axis (other than infinity) is the net \vec{E}_{net} of the sheet and particle is zero?

$\vec{E}_{\text{net}} = \text{zero}$ in A or B regions
impossible $\vec{E}_{\text{net}} = \text{zero}$ in C region



$$\vec{E}_{\text{sheet}} = \frac{\sigma}{2\epsilon_0} = -\frac{2 \times 10^{-6}}{2\epsilon_0} = \frac{-10^{-6}}{\epsilon_0} \frac{\text{C}}{\text{m}^2}$$

$$\Rightarrow \vec{E}_{\text{sheet}} = \vec{E}_{\text{sphere}}$$

$$\frac{\sigma}{2\epsilon_0} = \frac{kQ}{x^2} = \frac{1}{4\pi\epsilon_0} \frac{Q}{x^2}, \quad Q = 8\mu\text{C}$$

$$x^2 = \frac{2}{\pi}, \quad x = \sqrt{\frac{2}{\pi}} = \sqrt{0.637} = 0.798$$

$$d = 0.2 \text{ m}, \quad |0.798| > 0.2$$

So a) 0.798 m

b) -0.798 m

c) if $d = 0.950 \text{ m}$ at what coordinate on the x-axis is $\vec{E}_{\text{net}} = 0$?

if $d = 0.950$

$$d > | -0.798 |$$

$0.950 > 0.798$ it is not possible to get a negative coordinate.

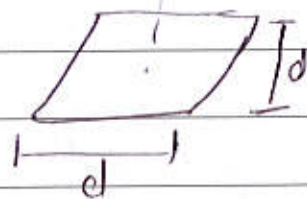
So the only coordinate in x-axis is 0.798 m

[17] A proton is a distance $\frac{d}{2}$ directly above the center of a square of side d . What is the electric flux through a square? (Think of the square as one of a cube with edge d)

$$\Phi = \Phi_{\text{total}} / 6$$

$$\Phi_{\text{total, cube}} = q / \epsilon_0 = 1.6 \times 10^{-19} / 8.85 \times 10^{-12}$$

$$= 1.807 \times 10^{-8} \frac{\text{N}\cdot\text{m}^2}{\text{C}}$$



$$\Phi = 3.01 \times 10^{-9} \frac{\text{N}\cdot\text{m}^2}{\text{C}}$$

one surface of cube

[18] A long, nonconducting, solid cylinder of radius 4.0 cm has a non uniform volume charge density ρ that is a function of radial distance r from the cylinder axis

$$\rho = A r^2, \quad A = 6.3 \text{ } \mu\text{C}/\text{m}^3$$

What is the magnitude of the electric field at $r = 3 \text{ cm}$ and 5 cm ?

$$\Phi = \frac{q_{\text{enc}}}{\epsilon_0}$$



$$R = 0.04 \text{ m}$$

$$\rho = A r^2, \quad A = 6.3 \text{ } \mu\text{C}/\text{m}^3$$

$$\ast \quad r = 3.00 \text{ cm} = 0.03 \text{ m}$$

$$V = \pi r^2 h$$

$$dV = 2\pi r h dr$$

$$Q_{\text{total}} = \int_0^R \rho dV$$

$$Q = \int \rho dV = \int A r^2 2\pi r h dr = 2\pi h A \frac{r^4}{4}$$

$$\text{if } Q_{\text{total}} = \int_0^R \rho dV = 2\pi h A \frac{R^4}{4} = 2.5 \times 10^{-11} h$$

$$\Rightarrow q_{\text{enc}} = \frac{\pi h A r^4}{2}$$

$$\rightarrow \text{Gauss' Law} \Rightarrow \Phi = \frac{q_{\text{enc}}}{\epsilon_0} = E(A)$$

$$E = \frac{\pi h A r^4}{2\epsilon_0} \frac{1}{2\pi r h}$$

$$E = \frac{A r^3}{4\epsilon_0}$$

$$E(3\text{cm}) = \frac{6.3 \times 10^{-6} (0.03)^3}{8.85 \times 10^{-12} \times 4} \sim 4.8 \text{ N/C}$$

• $r = 5\text{cm}$, $r > R$

$$q_{\text{total}} = 2.5 \times 10^{-11} \text{ h}$$

$$E(2\pi r h) = \frac{q_{\text{total}}}{\epsilon_0} \Rightarrow \frac{2.5 \times 10^{-11} \text{ h}}{2\pi r h \epsilon_0}$$

$$E = 8.496 \text{ N/C}$$

$$\lambda = \frac{q}{h} = 2.5 \times 10^{-11}$$

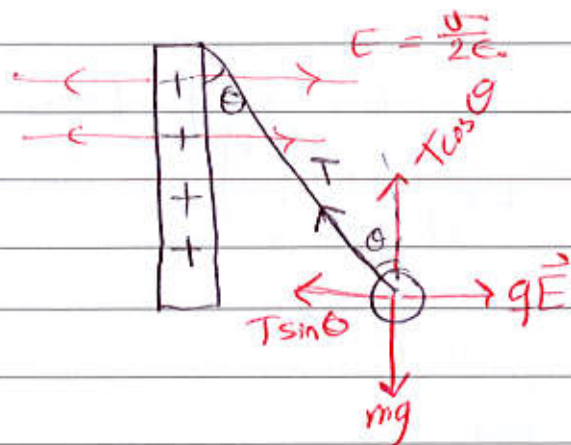
$$\lambda = \frac{\pi}{2} A r^4 ; (r=R), \lambda = 2.5 \times 10^{-11} \text{ C/m}$$

$$E = \frac{\lambda}{2\pi \epsilon_0 r} \text{ H}$$

[21] A small non conducting ball of mass $m = 7.3 \text{ mg}$ and charge $q = 2 \times 10^{-8} \text{ C}$ (distributed uniformly through its volume) hangs from an insulating thread that makes an angle $\theta = 30^\circ$ with a vertical, uniformly charged non-conducting sheet. Considering the gravitational force on the ball and assuming the sheet extends far vertically and into and out of the page, calculate the surface charge density σ of the sheet? $\sigma = ??$

$$\begin{aligned} T \sin \theta &= qE \\ T \cos \theta &= mg \Rightarrow T = \frac{mg}{\cos \theta} \\ \rightarrow \frac{mg \sin \theta}{\cos \theta} &= qE \end{aligned}$$

$$E = \frac{mg \tan \theta}{q} = \frac{\sigma}{2\epsilon_0}$$



$$\sigma = \frac{mg \tan \theta}{q} \cdot 2 \epsilon_0$$

$$= \frac{7.3 \times 10^{-6} \times 9.8 \times \tan 30 \times 2 \times 8.85 \times 10^{-12}}{2 \times 10^{-8}}$$

$$= 3.66 \times 10^{-8} \text{ C/m}^2 = 37 \text{ nC/m}^2$$

[28] A spherical shell with uniform volume charge density $\rho = 1.56 \text{ nC/m}^3$, inner radius $a = 10.0 \text{ cm}$ and outer radius $b = 2.00 a$. What is the magnitude of the \vec{E} at radial distances?

a) $r = 0$

spherical shell $E = \begin{cases} \frac{kq}{r^2} & r > R \\ 0 & r < R \end{cases}$

$E(r=0) = 0$ inside the spherical shell.



b) $r = \frac{a}{2.00}$, $E(r = \frac{a}{2}) = 0$
 $\frac{a}{2} < a$ inner radius.

c) $r = a$, $E(r = a) = 0$

For $a \leq r \leq b \Rightarrow E = \frac{k q_{enc}}{r^2}$, $\rho = \frac{q_{enc}}{V}$

$$E = \frac{k\rho}{r^2} \left[\frac{4\pi r^3}{3} - \frac{4\pi a^3}{3} \right], \quad a \leq r \leq b \quad q_{enc} = \rho V$$

at $r = a \Rightarrow E = \frac{k\rho}{a^2} \left[\frac{4\pi a^3}{3} - \frac{4\pi a^3}{3} \right] = 0$

d) $r = 1.5a$

$$E = \frac{k\rho}{r^2} \left[\frac{4\pi r^3}{3} - \frac{4\pi a^3}{3} \right] = \frac{\rho}{4\pi\epsilon_0 r^2} \frac{4\pi}{3} [r^3 - a^3]$$

$$E = \frac{\rho}{3\epsilon_0 r^2} [r^3 - a^3]$$

$$E(r=1.5a) = \frac{\rho}{3\epsilon_0 (1.5)^2 a^2} [a^3 (1.5)^3 - a^3] = 6.2 \text{ N/C}$$

e) $r = b = 2a$

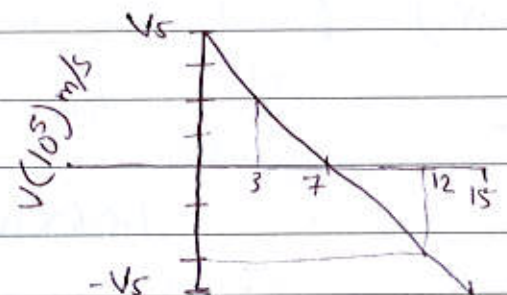
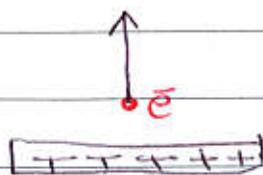
$$E = \frac{\rho}{3\epsilon_0 (2a)^2} [(2a)^3 - a^3] = \frac{7\rho a}{3\epsilon_0 4} = 10.28 \text{ N/C}$$

f) $r = 3b$, $r \geq b$ $E = \frac{q_{\text{total}}}{4\pi\epsilon_0 r^2}$, $r = 6a$

$$E = \frac{\rho}{3\epsilon_0} \left[\frac{b^3 - a^3}{r^2} \right] = \frac{\rho}{3\epsilon_0} \left(\frac{(2a)^3 - a^3}{(6a)^2} \right)$$

$$= \frac{\rho}{3\epsilon_0} \frac{7a^3}{36a^2} = \frac{7\rho a}{3 \times 36 \epsilon_0} = 1.14 \text{ N/C}$$

[2] An electron is shot directly away from a uniformly charged plastic sheet, at speed $v_s = 1.6 \times 10^5 \text{ m/s}$. The sheet is nonconducting flat and very large.



Sheet's surface charge density?

electron's vertical velocity component versus t until the return to launch point.

$$a = \text{slope } (V \text{ vs } t) \quad (0, 1.6 \times 10^5), (7, 0)$$

$$a = \frac{0 - 1.6 \times 10^5}{(7-0) \times 10^{-12}} = -2.29 \times 10^{16} \text{ m/s}^2$$

$$F = qE = ma$$

$$q \frac{V}{2E} = ma$$

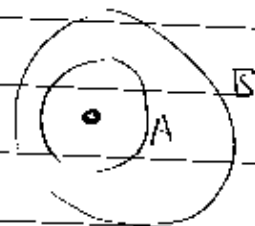
$$\sigma = \frac{2ma \epsilon_0}{q} = \frac{2(9.1 \times 10^{-31} \text{ kg})(-2.29 \times 10^{16})}{1.6 \times 10^{-19}} (8.85 \times 10^{-12})$$

$$= -2.3 \times 10^{-6} \text{ C/m}^2$$

[54] Flux and nonconducting shells, A charged particle is suspended at the center of two concentric spherical shells. They are very thin and made of nonconducting material.

a) What is the charge of the central particle?

b) What are the net charges of shell A and B?



$$\epsilon_0 \oint \vec{E} = q_{enc}$$

a) for small r , $\vec{\Phi} = 4 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}$

$$q_{enc} = 8.85 \times 10^{-12} \times 4 \times 10^5 = 3.54 \times 10^{-6} \text{ C}$$

b) $\vec{\Phi} = -8 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}$, $q_{enc} = -7.08 \times 10^{-6} \text{ C}$

$$\frac{q}{A} = q_{enc} = q_{\text{central particle}} = -7.08 \times 10^{-6} - 3.54 \times 10^{-6}$$

$$q_A = -10.62 \mu\text{C}$$

c) $\vec{\Phi} = 12 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}$, $q_{\text{total enclosed}} = 1.06 \times 10^{-5} \text{ C}$

$$\frac{q}{B} = q_{\text{total enc}} - q_A = q_{\text{central particle}} =$$

$$1.06 \times 10^{-5} + 7.08 \times 10^{-6} = 1.77 \times 10^{-5} \text{ C}$$

$$17.7 \mu\text{C}$$

ff

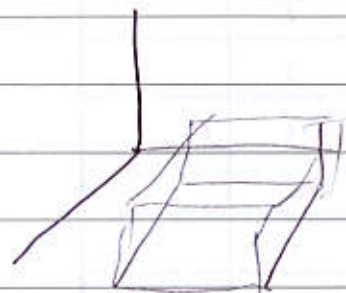
[34] The box like Gaussian surface encloses a net charge of $+32\epsilon_0 \text{ C}$ and lies in an electric field $\vec{E} = [(10+2x)\hat{i} - 3\hat{j} + bz\hat{k}] \text{ N/C}$
 x, z in meters. The bottom face is in the xz plane; the top face is in the horizontal plane passing through $y_2 = 1.00 \text{ m}$. For $x_1 = 1 \text{ m}, x_2 = 4 \text{ m}, z_1 = 1 \text{ m}, z_2 = 3 \text{ m}$. What is b ?

\vec{E} is constant in Y -direction, $\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$

• Φ through the two faces parallel to the xz plane = zero, since \vec{E} is constant in Y -direction

$$\Phi_{xz} = \iint [E_y(y_2) - E_y(y_1)] dx dz$$

$$= \int_1^4 dx \int_1^3 dz (-3 - (-3)) = 0$$



$$\Phi_{yz} = \iint [E_x(x_2) - E_x(x_1)] dy dz = \int_0^1 dy \int_1^3 dz [10 + 2(4) - 10 - 2(1)]$$

$$= 6 \int_0^1 dy \int_1^3 dz = 6(1)(2) = 12$$

$$\Phi_{xy} = \iint [E_z(z_2) - E_z(z_1)] dx dy = \int_1^4 dx \int_0^1 dy (3b - b)$$

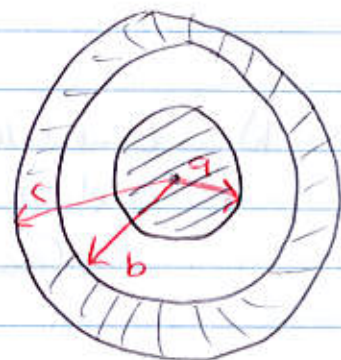
$$= (2b)(3)(1) = 6b$$

$$\Rightarrow \Phi_{\text{total}} = \frac{q_{\text{enc}}}{\epsilon_0} \Rightarrow 0 + 12 + 6b = \frac{+32\epsilon_0}{\epsilon_0}$$

$$b = 3.33 \text{ N/C}\cdot\text{m}$$

29

A solid sphere of radius $a = 2 \text{ cm}$ is concentric with a spherical conducting shell of inner radius $b = 2a$ and outer radius $c = 2.4a$. The sphere has a net uniform charge $q_1 = +2 \text{ fC}$. The shell has a net charge $q_2 = -q_1$. What is the magnitude of the electric field at radial distances - a) $r = 0$, b) $r = \frac{a}{2}$, c) $r = a$, d) $r = 1.5a$, e) $r = 2.3a$, f) $r = 3.5a$? What is the net charge on the g) inner and h) outer surface of the shell?



\vec{E} is radially outward.
 $\rightarrow \oint \vec{E} \cdot d\vec{A} = 4\pi r^2 E$

For $r < a$, $E = \frac{q_1 r}{4\pi\epsilon_0 a^3}$

a) $r = 0$, $E = 0$

b) $r = a/2$, $E = \frac{q_1 a/2}{4\pi\epsilon_0 a^3} = \frac{9 \times 10^9 \times 2 \times 10^{-15}}{2 (2 \times 10^{-2})^2} = 0.0225 \frac{\text{N}}{\text{C}}$

c) $r = a$, $E = \frac{q_1}{4\pi\epsilon_0 a^2} = \frac{9 \times 10^9 \times 2 \times 10^{-15}}{(2 \times 10^{-2})^2} = 0.045 \frac{\text{N}}{\text{C}}$

For $a < r < b$, $E = \frac{q_1}{4\pi\epsilon_0 r^2}$

d) $r = 1.5a$, $E = \frac{kq_1}{r^2} = \frac{9 \times 10^9 \times 2 \times 10^{-15}}{(1.5 \times 2 \times 10^{-2})^2} = 0.02 \frac{\text{N}}{\text{C}}$

e) In the region $b < r < c$, since the shell is conducting, the Electric field is zero, $E(2.3a) = 0$

f) $r > c$, the charge enclosed by the Gaussian surface is zero
 $E(r = 3.5a) = 0$

g) Gaussian surface that lies completely within the conducting shell
 $\vec{E} = 0$ every where on the surface
$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} = 0$$

So $q_{enc} = 0$, let $Q_{inner shell}$

$$q_1 + Q_{inner shell} = 0, \quad Q_{inner} = -q_1 \\ = -2 \mu C$$

h) $Q_{out shell} = 0$

net charge on the shell = $-q_1$

$$Q_{out} = -q_1 - Q_{inner} = 0$$